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MEMBRANE ANALYSIS OF PRESSURIZED THIN SPHEROID SHELLS COMPOSED OF FLAT GORES, AND ITS APPLICATION TO ECHO II

by Hossein Bahiman

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SUMMARY

Membrane analysis of very thin pressurized spheroid shells composed of flat gores led to a set of nonlinear differential equations for three membrane displacements. An approximate solution by the perturbation technique is established. As an example, the approximate shape of Echo II is computed.

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MEMBRANE ANALYSIS OF PRESSURIZED THIN SPHEROID SHELLS COMPOSED OF FLAT GORES, AND ITS APPLICATION TO ECHO II

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INTRODUCTION

To compute the shape of a pressurized thin spheroid shell composed of many identical flat gores requires solution of five nonlinear partial differential equations of equilibrium and boundary conditions for three displacements of a gore. The position of each point on the gore could be determined as a function of two independent variables, namely the point's distance from the center line of the gore and latitude from the equator of the spheroid. For an approximate solution to these nonlinear partial differential equations each displacement is expanded into a power series in the point's distance from the gore center line, such that the series coefficients are at most functions of the latitude. By equating to zero the terms of lowest power in x in each equation, five new nonlinear differential equations for the six coefficients (two for each displacement) are obtained. The approximate solution of these equations by the perturbation technique led to three displacement functions of the gore, and one unknown constant which was evaluated by applying the variational method to the total elastic strain and potential energy of the system.

In this analysis, it is assumed that the displacement of the spheroid due to gravity is negligible compared to that caused by the constant internal pressure. It is also assumed that the material of the spheroid is homogeneous and isotropic and it obeys Hooke's law.

Finally, this method of analysis is applied to Echo II, the 135-foot diameter passive communication satellite. However, since the 3-ply aluminum-Mylar-aluminum material of the satellite does not satisfy the aforementioned assumptions, the analytical results obtained here are expected to be only an approximation of the actual case.

THEORY

To determine the shape of a thin spheroid shell composed of identical flat gores it is sufficient to compute the true shape of a gore in the pressurized state. To do this, it is assumed that the spheroid material is homogeneous, isotropic, and linearly elastic. It is also assumed that the effect of gravity on the spheroid shape is negligible compared with that of the constant internal pressure.

Let rectangular Cartesian coordinate axes (Figure 1) be drawn so that the origin 0 is located at the center of the balloon, the y axis is along the polar axis, and the z axis is extended through the center of a gore. Draw $x' y' z'$ axes from a point P of the gore in such a way that the x' axis will be parallel to the x axis, the z' axis will be in the direction OP, and the y' (or θ) axis will be tangent to the meridian at point P.

In the pressurized state of equilibrium, the resultant of all the forces acting on an element of the shell in the direction x', θ and z' vanishes. Let N_x, N_θ, and $N_{x\theta}$ be normal and shear tractions per unit length of membrane, θ be the angle between the position vector and the equatorial plane, and p be the internal pressure of the spheroid (Figure 2). Let u, v, and w be displacement components at a point P in the directions x', y', and z' respectively. Then (Reference 1)

$$\left.\begin{aligned} \frac{\partial N_x}{\partial x} + \frac{\partial N_{x\theta}}{r\partial\theta} &= 0 \ , \\ \frac{\partial N_{x\theta}}{\partial x} + \frac{\partial N\theta}{r\partial\theta} &= 0 \ , \\ \frac{N_\theta}{r} - N_\theta \frac{\partial^2 w}{r^2\,\partial\theta^2} - N_x \frac{\partial^2 w}{\partial x^2} - 2N_{x\theta} \frac{\partial^2 w}{r\partial x\partial\theta} &= p \ . \end{aligned}\right\} \tag{1}$$

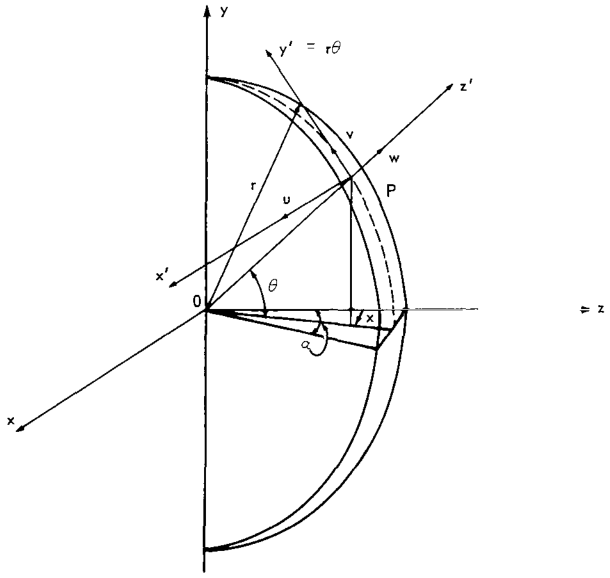

Figure 1—Coordinate axes and displacement components.

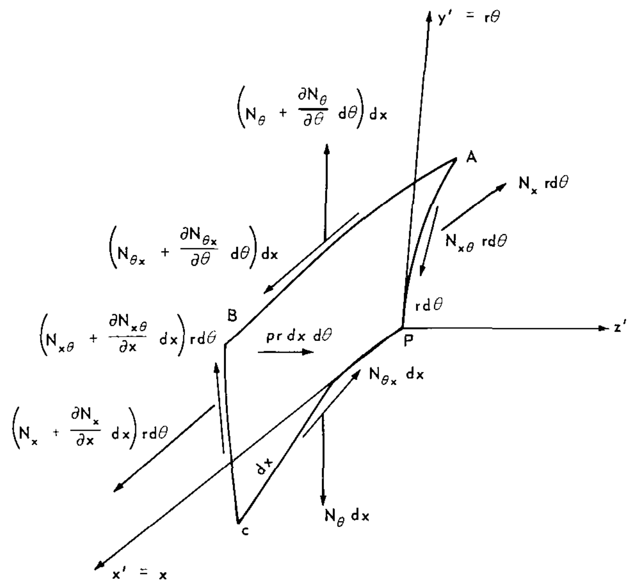

Figure 2—Forces acting on an element of a gore.

The elastic strains are given by:

$$
\left.\begin{aligned}
\epsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 , \\
\epsilon_\theta &= \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{w}{r} + \frac{1}{2r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 , \\
\epsilon_{x\theta} &= \frac{1}{2}\left(\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} + \frac{1}{r}\frac{\partial w}{\partial x}\frac{\partial w}{\partial \theta}\right) .
\end{aligned}\right\} \tag{2}
$$

Also,

$$
\left.\begin{aligned}
N_x &= K\left(\epsilon_x + \mu\,\epsilon_\theta\right) , \\
N_\theta &= K\left(\epsilon_\theta + \mu\,\epsilon_x\right) , \\
N_{x\theta} &= K(1-\mu)\,\epsilon_{x\theta} ,
\end{aligned}\right\} \tag{3}
$$

where $K = Et/(1-\mu^2)$ is the membrane stiffness, and E, t, and μ are Young's modulus, the shell thickness, and Poisson's ratio, respectively. Substitution of Equations 2 into Equations 3 leads to

$$
\left.\begin{aligned}
N_x &= K\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \mu\frac{\partial v}{r\partial\theta} + \mu\frac{w}{r} + \frac{\mu}{2}\left(\frac{\partial w}{r\partial\theta}\right)^2\right] , \\
N_\theta &= K\left[\frac{\partial v}{r\partial\theta} + \frac{w}{r} + \frac{1}{2}\left(\frac{\partial w}{r\partial\theta}\right)^2 + \mu\frac{\partial u}{\partial x} + \frac{\mu}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] , \\
N_{x\theta} &= \frac{K(1-\mu)}{2}\left(\frac{\partial u}{r\partial\theta} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{r\partial\theta}\right) .
\end{aligned}\right\} \tag{4}
$$

And substitution of Equations 4 into Equations 1 yields

$$
\left.\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 v}{r\partial x\partial\theta} + \frac{\mu}{r}\frac{\partial w}{\partial x} + \frac{\mu}{r^2}\frac{\partial^2 w}{\partial x\partial\theta}\frac{\partial w}{\partial\theta} + \frac{1-\mu}{2r}\left(\frac{1}{r}\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 v}{\partial x\partial\theta} + \frac{1}{r}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial\theta^2} + \frac{1}{r}\frac{\partial^2 w}{\partial x\partial\theta}\frac{\partial w}{\partial\theta}\right) = 0 , \\
&\frac{1-\mu}{2}\left(\frac{1}{r}\frac{\partial^2 u}{\partial x\partial\theta} + \frac{\partial^2 v}{\partial x^2} + \frac{1}{r}\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial\theta} + \frac{1}{r}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial\theta}\right) + \frac{\partial^2 v}{r^2\partial\theta^2} + \frac{1}{r^2}\frac{\partial w}{\partial\theta} + \frac{1}{r^3}\frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial\theta^2} + \frac{\mu}{r}\frac{\partial^2 u}{\partial x\partial\theta} \\
&\qquad - \frac{\mu}{r}w\frac{\partial^3 w}{\partial x^2\partial\theta} - \frac{\mu}{r}\frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial x^2} + \frac{\mu}{r}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial\theta} = 0 , \\
&\left[\frac{1}{r^2}\frac{\partial v}{\partial\theta} + \frac{w}{r^2} + \frac{1}{2r}\left(\frac{\partial w}{r\partial\theta}\right)^2 + \frac{\mu}{r}\frac{\partial u}{\partial x} - \frac{\mu}{r}w\frac{\partial^2 w}{\partial x^2} + \frac{\mu}{2r}\left(\frac{\partial w}{\partial x}\right)^2\right]\left(1 - \frac{1}{r}\frac{\partial^2 w}{\partial\theta^2}\right) - \frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2} \\
&\quad - \frac{\mu}{r}\frac{\partial v}{\partial\theta}\frac{\partial^2 w}{\partial x^2} - \frac{\mu}{r}w\frac{\partial^2 w}{\partial x^2} - \frac{\mu}{r^2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial\theta}\right)^2 - (1-\mu)\left(\frac{1}{r^2}\frac{\partial u}{\partial\theta}\frac{\partial^2 w}{\partial x\partial\theta} + \frac{1}{r}\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x\partial\theta} + \frac{1}{r^2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial x\partial\theta}\right) = \frac{p}{K} .
\end{aligned}\right\} \tag{5}
$$

The boundary conditions are:

$$\left.\begin{aligned} N_{x\theta} &= 0 && \text{on} \quad x = r\alpha\cos\theta \\ u &= \alpha w\cos\theta - \alpha v\sin\theta && \text{on} \quad x = r\alpha\cos\theta \end{aligned}\right\} \tag{6}$$

along the seam, where $\alpha = \pi/n$. The first expression indicates that because of symmetry shear stress vanishes along the seam where any two adjacent gores meet; and the second expression implies that the points along each seam move in the meridional plane. Since the coefficients of only the first two terms of the power-series expansion of displacements will be determined by this method, continuity of slope in the plane normal to the seam cannot be used as a third boundary condition. The use of such a boundary condition would lead to an erroneous solution, namely, that the cross section of the spheroid with any plane normal to its polar axis is a true circle, no matter what the magnitude of the internal pressure. Instead, a virtual displacement is given to the pressurized spheroid in the state of equilibrium and change of the total elastic strain and potential energy of the spheroid and the load is equated to zero.

Let r be the radius of the centerline of a gore before any load is applied, and a, b, c, d, f and g be functions of the variable angle θ. Displacements w, u, and v may be expanded into power series as follows:

$$\left.\begin{aligned} w &= ar + b\frac{x^2}{r} + \cdots , \\ u &= cx + d\frac{x^3}{r^2} + \cdots , \\ v &= fr + g\frac{x^2}{r} + \cdots . \end{aligned}\right\} \tag{7}$$

Substitution of Equations 7 and the last of Equations 4 into Equations 5 and 6 leads to

$$\left.\begin{aligned} & 6d + (1+\mu)\frac{dg}{d\theta} + 2\mu b + (1+\mu)\frac{da}{d\theta}\frac{db}{d\theta} + \frac{1-\mu}{2}\frac{d^2 c}{d\theta^2} + (1-\mu)\, b\,\frac{d^2 a}{d\theta^2} + 4b^2 = 0 , \\ & \frac{1+\mu}{2}\frac{dc}{d\theta} + (1-\mu)\, g + (1-3\mu)\, b\,\frac{da}{d\theta} + \frac{d^2 f}{d\theta^2} + \frac{da}{d\theta} + \frac{da}{d\theta}\frac{d^2 a}{d\theta^2} - 2\mu a\frac{db}{d\theta} = 0 , \\ & \left[\frac{df}{d\theta} + a + \frac{1}{2}\left(\frac{da}{d\theta}\right)^2 + \mu c - 2\mu ab\right]\left(1 - \frac{d^2 a}{d\theta^2}\right) - 2bc - 2\mu b - 2\mu ab - \mu b\left(\frac{da}{d\theta}\right)^2 = \frac{pr}{K} , \\ & \frac{dc}{d\theta} + 2g + 2b\frac{da}{d\theta} = 0 , \\ & c - a + f\tan\theta = (b-d)\,\alpha^2\cos^2\theta . \end{aligned}\right\} \tag{8}$$

Neglecting small terms of higher order in Equations 8 leads to

$$\left.\begin{aligned}
6d + 2\mu b - \mu \frac{d^2 c}{d\theta^2} - 2\mu b \frac{d^2 a}{d\theta^2} + 4b^2 &= 0 , \\
\mu \frac{dc}{d\theta} - 2\mu b \frac{da}{d\theta} + \frac{d^2 f}{d\theta^2} + \frac{da}{d\theta} - 2\mu a \frac{db}{d\theta} &= 0 , \\
\frac{df}{d\theta} + a + \mu c - 4\mu ab - 2bc - 2\mu b \frac{df}{d\theta} &= \frac{pr}{K} , \\
c - a + f \tan\theta &= (b-d)\,\alpha^2 \cos^2\theta , \\
g &= -\frac{1}{2}\frac{dc}{d\theta} - b \frac{da}{d\theta} .
\end{aligned}\right\} \tag{9}$$

Since n, the number of gores, is assumed to be large, α^2 is very small. Thus the perturbation technique can be applied to the functions a, b, c, d, f, and g in the following fashion:

$$\left.\begin{aligned}
a(\theta) &= a_0(\theta) + \alpha^2 a_1(\theta) + \cdots , \\
b(\theta) &= b_0(\theta) + \alpha^2 b_1(\theta) + \cdots , \\
c(\theta) &= c_0(\theta) + \alpha^2 c_1(\theta) + \cdots , \\
d(\theta) &= d_0(\theta) + \alpha^2 d_1(\theta) + \cdots , \\
f(\theta) &= f_0(\theta) + \alpha^2 f_1(\theta) + \cdots , \\
g(\theta) &= g_0(\theta) + \alpha^2 g_1(\theta) + \cdots .
\end{aligned}\right\} \tag{10}$$

Substitution of Equations 10 into Equations 9 yields a set of five nonlinear differential equations in terms of a_0, b_0, c_0, d_0, f_0, g_0, a_1, b_1, c_1, d_1, etc. containing even powers of α. Terms containing each power of α in each differential equation must independently vanish. The terms independent of α must vanish in the following way:

$$\left.\begin{aligned}
6d_0 + 2\mu b_0 - \mu \frac{d^2 c_0}{d\theta^2} - 2\mu b \frac{d^2 a_0}{d\theta^2} + 4b_0^2 &= 0 , \\
\mu \frac{dc_0}{d\theta} - 2\mu b_0 \frac{da_0}{d\theta} + \frac{d^2 f_0}{d\theta^2} + \frac{da_0}{d\theta} - 2\mu a_0 \frac{db_0}{d\theta} &= 0 , \\
\frac{df_0}{d\theta} + a_0 + \mu c_0 - 4\mu a_0 b_0 - 2b_0 c_0 - 2\mu b_0 \frac{df_0}{d\theta} &= 0 , \\
c_0 - a_0 + f_0 \tan\theta &= 0 , \\
g_0 &= -\frac{1}{2}\frac{dc_0}{d\theta} - b_0 \frac{da_0}{d\theta} .
\end{aligned}\right\} \tag{11}$$

The solution of the above equations could be given as the following:

$$\left.\begin{aligned} b_0 &= \text{constant} = B\,, \\ d_0 &= -\frac{2}{3}B^2 - \frac{\mu}{3}B\,, \\ a_0 &= c_0 = f_0 = g_0 = 0\,. \end{aligned}\right\} \qquad (12)$$

The coefficients of α^2 in Equations 9 must vanish in the following fashion:

$$\left.\begin{aligned} & 6d_1 + 2\mu b_1 - \mu\frac{d^2 c_1}{d\theta^2} - 2\mu B\frac{d^2 a_1}{d\theta^2} + 8Bb_1 = 0\,, \\ & \mu\frac{dc_1}{d\theta} - 2\mu B\frac{da_1}{d\theta} + \frac{d^2 f_1}{d\theta^2} + \frac{da_1}{d\theta} = 0\,, \\ & \frac{df_1}{d\theta} + a_1 + \mu c_1 - 4\mu B a_1 - 2Bc_1 - 2\mu B\frac{df_1}{d\theta} = \frac{pr}{K\alpha^2}\,, \\ & c_1 - a_1 + f_1\tan\theta = \left[\frac{2}{3}B^2 + \left(1 + \frac{\mu}{3}\right)B\right]\cos^2\theta\,, \\ & g_1 = -\frac{1}{2}\frac{dc_1}{d\theta} - B\frac{da_1}{d\theta}\,. \end{aligned}\right\} \qquad (13)$$

The solution of Equations 13 could be written as:

$$\left.\begin{aligned} a_1 &= A_1\cos 2\theta + A_2\,, \\ b_1 &= 0\,, \\ c_1 &= C_1\cos 2\theta + C_2\,, \\ d_1 &= D_1\cos 2\theta\,, \\ f_1 &= F_1\sin 2\theta\,, \\ g_1 &= G_1\sin 2\theta\,, \end{aligned}\right\} \qquad (14)$$

in which the constant coefficients are given by

$$
\left.
\begin{aligned}
\gamma &= \frac{1}{3} B^2 + \left(\frac{1}{2} + \frac{\mu}{3}\right) B \ , \\
F_1 &= \frac{\left(2\mu B + 1 + \mu^2\right) \gamma}{-4\mu^2 B - 2\mu B + \mu^2 - 1} \ , \\
A_1 &= \frac{\mu\gamma + (2 + \mu) F_1}{2\mu B - \mu - 1} \ , \\
C_1 &= \gamma + A_1 + F_1 \ , \\
A_2 &= \frac{(\mu - 2B)\left(F_1 - \gamma\right) + \dfrac{pr}{K\alpha^2}}{1 + \mu - 4\mu B - 2B} \ , \\
C_2 &= A_2 - F_1 + \gamma \ , \\
D_1 &= -\frac{2\mu}{3} C_1 - \frac{4}{3} \mu B A_1 \ , \\
G_1 &= C_1 + 2BA_1 \ .
\end{aligned}
\right\} \tag{15}
$$

Knowing the value of Poisson's ratio, one can easily evaluate in an orderly manner γ, F_1, A_1, C_1, A_2, C_2, D_1, and G_1 as a function of the Constant B.

To evaluate constant B, the method of virtual displacement may be utilized. The elastic strain energy of one quarter of a gore is given by

$$
T = \int_{\theta=0}^{\theta=\pi/2} \int_{x=0}^{x=r\alpha\cos\theta} \frac{1}{2} \left(N_x \epsilon_x + N_\theta \epsilon_\theta + 2N_{x\theta} \epsilon_{x\theta}\right) r \, dx \, d\theta \ . \tag{16}
$$

Substitution of stress and strain values in Equation 16 yields

$$
\begin{aligned}
T = \int_{\theta=0}^{\theta=\pi/2} \int_{x=0}^{x=r\alpha\cos\theta} \frac{K}{2} \Bigg[& \left(c + 3d \frac{x^2}{r^2} - 2ab\right)^2 + \left(\frac{df}{d\theta} + \frac{x^2}{r^2} \frac{dg}{d\theta} + a + b \frac{\alpha^2}{r^2} + \frac{1}{2} \left(\frac{da}{d\theta}\right)^2 + \frac{1}{2} \frac{x^4}{r^4} \left(\frac{db}{d\theta}\right)^2 \right. \\
& \left. + \frac{x^2}{r^2} \frac{da}{d\theta} \frac{db}{d\theta}\right)^2 + 2\mu \left(c + 3d \frac{x^2}{r^2} - 2ab\right) \left(\frac{df}{d\theta} + \frac{x^2}{r^2} \frac{dg}{d\theta} + a + b \frac{x^2}{r^2} + \frac{1}{2} \left(\frac{da}{d\theta}\right)^2 + \frac{1}{2} \frac{x^4}{r^4} \left(\frac{db}{d\theta}\right)^2 \right. \\
& \left. + \frac{x^2}{r^2} \frac{da}{d\theta} \frac{db}{d\theta}\right) + \frac{1 - \mu}{2} \left(\frac{x}{r} \frac{dc}{d\theta} + \frac{x^3}{r^3} \frac{dd}{d\theta} + \frac{2x}{r} g + \frac{2x}{r} b \frac{da}{d\theta} + 2 \frac{x^3}{r^3} b \frac{db}{d\theta}\right)^2 \Bigg] r \, dx \, d\theta \ .
\end{aligned} \tag{17}
$$

In a virtual displacement corresponding to change of B by δB the work done by the internal pressure for a quarter of a gore is given by

$$\frac{\partial \overline{W}}{\partial B} \delta B = p \int_{\theta=0}^{\theta=\pi/2} \int_{x=0}^{x=r\alpha\cos\theta} \frac{\partial w}{\partial B} r \, dx \, d\theta \, \delta B , \tag{18}$$

in which $\overline{W}$ is the total work done by the internal pressure for a quarter gore, and w is the radial displacement given by Equations 7. Hence, the Constant B could be determined from

$$\frac{\partial T}{\partial B} = p \int_{\theta=0}^{\theta=\pi/2} \int_{x=0}^{x=r\alpha\cos\theta} \frac{\partial w}{\partial B} r \, dx \, d\theta . \tag{19}$$

Thus for a given pressure p, Equation 19 in conjunction with Equations 17, 7, 10, 12, 14 and 15 furnishes B, which upon substitution into Equations 15, gives coefficients A_1, A_2, etc. Substitution of these coefficients into Equations 14 in conjunction with Equations 7, 10 and 12 gives the three displacement functions u, v, and w.

CALCULATION FOR ECHO II

The 135-foot diameter Echo II is composed of 106 identical gores with a maximum width of 4 feet which occurs in the equatorial plane of the balloon. The 3-ply material of the balloon is composed of 0.00035-inch thick Mylar sandwiched between two layers of aluminum each 0.00018-inch thick. The adjoining gores are located edge-to-edge with one-inch wide tape of the same material sealing the seams lengthwise. When separation of the two half-canisters takes place in the initial phase of orbiting, the residual air and water vapor inside the *folded* balloon—which is of the order of 1 mm Hg—inflates the satellite. While initially flat, each gore forms a narrow transverse portion of a half circular cylindrical surface 135 feet in diameter. Thus the initial shape of the balloon is a spheroid with 0.356-inch maximum deviation from a sphere which occurs in the equatorial plane.

Because of solar radiation the skin temperature increases and as a result inflation material sublimes inside the *open* balloon, building up internal pressure until a maximum of 225 μ Hg is reached; this maximum pressure corresponds to a membrane stress of 1.7 lb/in.

Although the material of the balloon is rather nonhomogeneous and anisotropic and does not quite obey Hooke's law, for the purpose of approximation it may be assumed that the requirements of homogeneity, isotropy and linear elasticity necessitated by this analysis are met. The average value of K corresponding to a skin traction of 1.70 lb/in. (given by Tests 1, 3, 4, and 6 of Reference 2) is 1050 lb/in.

Since nominal Young's modulus for aluminum is 10.5×10^6 psi and for Mylar is 0.55×10^6 psi, therefore it is expected that aluminum should carry most of the load. The nominal value of K for aluminum layers is given by

$$K_n = \frac{Et}{1 - \mu^2} = 4240 \text{ lb/inch} ,$$

in which $\mu = 0.33$ has been inserted. This value of K_n is much larger than the experimental value $K = 1050$ lb/in. Therefore, the actual Young's modulus and perhaps Poisson's ratio for thin layers of aluminum are smaller than the nominal ones.

Following the steps suggested in the last paragraph of the previous section, one could get an equation for B whose coefficients are functions of parameter $m = pr/Ka^2$. For different values of maximum pressure B is computed and listed in Table 1. Hence for $p = 225\,\mu$ Hg (or 0.000435 lb/in^2)

$$m = \frac{0.000435 \times 810}{1050 \left(\frac{\pi}{106}\right)^2} = 3.81 ,$$

Table 1

Values of B versus m.

m	B	m	B	m	B	m	B	m	B
0	0	2.0	-.294	4.0	-.320	6.0	-.320	8.0	-.317
.1	-.043	2.1	-.298	4.1	-.320	6.1	-.320	8.1	-.317
.2	-.079	2.2	-.300	4.2	-.320	6.2	-.320	8.2	-.317
.3	-.109	2.3	-.303	4.3	-.320	6.3	-.319	8.3	-.317
.4	-.135	2.4	-.305	4.4	-.320	6.4	-.319	8.4	-.317
.5	-.157	2.5	-.307	4.5	-.320	6.5	-.319	8.5	-.317
.6	-.176	2.6	-.309	4.6	-.320	6.6	-.319	8.6	-.316
.7	-.193	2.7	-.311	4.7	-.320	6.7	-.319	8.7	-.316
.8	-.207	2.8	-.312	4.8	-.321	6.8	-.319	8.8	-.316
.9	-.220	2.9	-.313	4.9	-.321	6.9	-.319	8.9	-.316
1.0	-.231	3.0	-.314	5.0	-.321	7.0	-.319	9.0	-.316
1.1	-.241	3.1	-.315	5.1	-.321	7.1	-.318	9.1	-.316
1.2	-.250	3.2	-.316	5.2	-.321	7.2	-.318	9.2	-.316
1.3	-.258	3.3	-.317	5.3	-.321	7.3	-.318	9.3	-.316
1.4	-.265	3.4	-.318	5.4	-.320	7.4	-.318	9.4	-.316
1.5	-.272	3.5	-.318	5.5	-.320	7.5	-.318	9.5	-.315
1.6	-.277	3.6	-.319	5.6	-.320	7.6	-.318	9.6	-.315
1.7	-.282	3.7	-.319	5.7	-.320	7.7	-.318	9.7	-.315
1.8	-.287	3.8	-.319	5.8	-.320	7.8	-.317	9.8	-.315
1.9	-.291	3.9	-.320	5.9	-.320	7.9	-.317	9.9	-.315

which corresponds to $B = -0.319$ in Table 1. Thus, the maximum differential radial displacement at the midgore and at the seam which occurs in the equatorial plane is given by

$$w\Big|_{\substack{\theta=0\\x=0}} - w\Big|_{\substack{\theta=0\\x=24^{in}}} = B\frac{x^2}{r}\Big|_{x=24^{in}} = 0.227^{in}$$

This corresponds to a maximum radial difference

$$\Delta = 0.356 - 0.227 = 0.129^{in}$$

in which $0.356^{in.}$ is in its initial unpressurized value.

CONCLUSIONS

The maximum differential radial displacement of a pressurized spheroid composed of identical flat gores, in a zero gravity field, as computed by this technique is believed to be a very good approximation of the true one, provided that the material is homogeneous, isotropic, and linearly elastic.

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